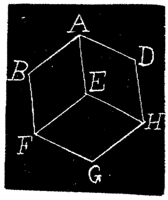
the position of DH or BF. The equations of DH are $x=\frac{1}{2}\sqrt{2}$, $y=-\sqrt{2}z$, and the equations of BF are $x=-\frac{1}{2}\sqrt{2}$, $y=-\sqrt{2}z$. In either case



$x^2=\frac{1}{2}$ and $y^2=2z^2$ and $x^2+y^2=2z^2+\frac{a^2}{2}$ which is the equation of the surface generated by the gauche hexagon $EHDCBF$. This surface could also be generated by the hyperbola $x^2=2z^2+\frac{1}{2}$. Hence the volume of the hyperboloid of one nappe generated$=\int \pi x^2 dz$, the upper limit being $\frac{1}{6}\sqrt{3}a$ and the lower limit $-\frac{1}{6}\sqrt{3}a$. This integral is $\frac{5}{27}\pi\sqrt{3}a^3$.

The lines AB, AE, and AD generate a cone, radius$=\frac{1}{3}\sqrt{6}a$, altitude$=\frac{1}{3}\sqrt{3}a$, volume$=\frac{2}{27}\pi\sqrt{3}a^3$.

The lines GF, GH, and GC generate another cone of the same size.

The sum of the volumes of the three solids$=\frac{1}{3}\pi\sqrt{3}a^3=1.8138a^3$.

[Note.—This solution by the Proposer is fuller than that given in the November number, and is published because several of our contributors failed to comprehend the abbreviated solution previously published. Prof. Whitaker asserts that the solution by Dr. Zerr in the September-October number is incorrect, while the latter says he does not as yet see Prof. Whitaker's hyperboloid. The above seems to be correct, but we shall be glad to have the criticisms of other contributors.—Editor.]

43. Proposed by Professor C. E. WHITE, A. M., Trafalgar, Indiana.

Prove that $\int_2^1 \frac{x^{a-1}+x^{-a}}{1+x}\frac{dx}{x}=\log(\tan\frac{a\pi}{2})$, when $a>0$ and <1.

[*Williamson's Integral Calculus, p. 154.*]

Comment by C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts.

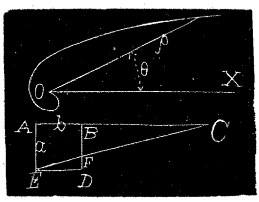
There seems to be an error in No. 43, as I find the following in my copy of *Williamson* :

$$\int_0^1 \frac{x^{a-1}+x^{-a}}{1+x}\,\frac{dx}{\log x},$$

which gives the required result.

[In *Williamson's Integral Calculus*, edition of 1891, the problem is given as published, but the mistake has doubtless been corrected in the later edition.—Editor.]

44. Proposed by DE VOLSON WOOD, C. E., Professor of Mechanical and Electrical Engineering in Stevens Institute of Technology, Hoboken. New Jersey.



Find the equation of a curve in which $\rho=f(\theta)$, in which ρ is equal to BC, an intercept of any secant drawn from the corner E of the rectangle $AEDB$, and prolonged to cut AB prolonged in C. Let equal increments of θ be proportional to the equal incrementsof DB as divided by the secant EF, θ being zero when EC coincides with ED, and $\theta=2\pi$ when EF passes through B. Determine the asymptotes.

I. Solution by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania; G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas; and ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University P. O., Mississippi.

Referring to the diagram given by the Proposer of this problem, July-Au-

gust (1895) MONTHLY, we have from the similar triangles FBC and FDE the following proportions: $BC : DE :: BF : DF$, or $\rho : b :: 2\pi - \theta : \theta$.

$\therefore (\rho+b)\theta = 2\pi b \ldots\ldots(1)$, which is the polar equation of *The Thistle of Scotland*, adopting the suggestion of Prof. MacCord.

Since $\rho^2(d\theta/d\rho) = -[(2\pi-\theta)^2/2\pi]b$, there is a *rectilinear asymptote* parallel to the initial line and at a distance $2\pi b$ *above* it. Making $\theta = \infty$, we have from (1) the equation $\rho = -b$; and this equation characterizes an *asymptotic circle* of radius b, or a *circular asymptote* of same radius, of the curve.

NOTE.—The derivation of (1) can be affected in, at least, *three* different ways; and, according to the conditions of the problem, (1) may also be written

$$(\rho+b)\theta = ab \ldots\ldots(2).$$

II. **Solution by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, P. O. Sebastopol, California.**

From figure given $\dfrac{BC}{ED} = \dfrac{BF}{DF} = \dfrac{BD-DF}{DF} = \dfrac{BD}{DF} - 1$,

or $\dfrac{\rho}{b} = \dfrac{2\pi}{\theta} - 1$; $\rho = \dfrac{2b\pi}{\theta} - b$, the equation of the curve.

When $\theta = 0$, $\rho = \infty$, and subtangent $= -2b\pi$.

The curve has, therefore, an asymptote parallel to OX at a distance above it, $2b\pi$, the circumference of a circle with radius AB.

The curve is concave toward the pole and intersects the axis perpendicularly and at a distance b to the left of the pole.

Elaborately solved by *O. W. ANTHONY*, and *C. W. M. BLACK*.

ERRATA.—On page 363, of last issue, line 4. omit $\sqrt{3}$ in the numerator of the second term; line 9, in the numerator, for "(a^2+x^2)" read (a^2-x^2); line 11, in the denominator of the second term, for "4" read 4^2; line 14, for "+" read =, before the last expression; page 364, line 15, for "of" read *to*; line 17, insert comma after "length"; line 17, for "$2n$" read 2π; line 18, for "π_2" read π^2; on same page, problem No. "43" should be No. 42; page 365, line 1, for "z^{n-2}" read z^{n-1}; and in line 2, of solution III., for "n^2+y^2" in the exponent, read x^2+y^2.

PROBLEMS.

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51. **Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

Find the maximum ellipsoid that can be cut out of a given right conic frustrum.

52. **Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.**

There are two lights of intensities m and n. Where must a target, whose surface is parallel to the line joining the two lights, be set up in order that it shall receive the maximum illumination per unit of area.